Connecting Geometry Unit

Kentucky Core Content: This unit has been designed following NCTM geometry standards for students in grades 6-8. Specifically targeting the following goals for students, each taken directly from http://standards.nctm.org

- precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties;
- create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.
- use visual tools such as networks to represent and solve problems;
- use geometric models to represent and explain numerical and algebraic relationships
- recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

Grade Level: 6-8

Teacher Background: This unit is designed for the middle school classroom with an emphasis on inquiry-based learning. It is meant to be an addendum to several weeks of instruction in geometry. To use this, students should already be familiar with polygons, the Pythagorean theorem, some probability and statistics, and nth term problems. I designed it with three goals in mind: 1) relating concrete concepts and ideas in geometry with real world problems, 2) having the student work with and solve "worthy mathematical tasks" and questions, through guided inquiry, and 3) relating geometry to other areas of mathematics.

Students' Preconceptions: These areas of mathematics relate to numerous student misconceptions. Some of those I came across when teaching the unit are included in the introductions for each of the individual lessons. The lesson on Atomic Origami addresses student misconceptions on the size of the atom and provides a framework for thinking of extremely small things in terms of scale. The Origami lesson also introduces the mobius strip, a figure which has intrigued mathematician for years because of the curious number of edges and sides. Many students think that the more times you flip a coin and receive tails means that you are more likely to get heads the next time when it has the same probability for each flip. In the Dizzy Darts lesson, it becomes clear that the probability of dropping a dart on the color blue depends solely on the size of the blue area as related to the size of the board. The Kongsberg Bridge problem begins with the misconception that every bridge may be crossed exactly once. Each of the lessons addresses a number of misconceptions through class discussion. At that point, it is up the teacher to discover these misconceptions and provide alternate explanations during "teachable moments".

Pre-Assessment: Other than class discussion, a teacher of this unit may want to design a pretest for their students. It should probably be open response and should cover all of the major areas of geometry covered by the unit as well as some of the related concept areas. Some pre-assessment also may include using an introductory lesson on the Pythagorean Theorem, counting and nth term problems, and simple event probability. Looking at student performance in these subjects, in particular gave me a good idea of where my students stood and what misconceptions they have about mathematics in a more general sense.

Evidence of Achievement: Every activity begins and ends with concepts taken from outside of the mathematics classroom. This helps the students relate math to the real world and gives them a framework to help understand more arbitrary mathematical concepts. This framework is the most immediate way to use guided inquiry for mathematics. Students given a worthy mathematical task will work to solve the problem, or answer the question given, using math reasoning, often without thinking of this as being math at all.

Once they have finished the problem the teacher can provide some foundation for the mathematics already accomplished. Throughout each of the activities provided, it often becomes useful or even necessary to provide students ways that the problems involve areas of mathematics that are directly related to Geometry. These include: Probability and Statistics, Combinatorics, Graphs (Networks), Number sense, and even Topology. Also, many of these activities involve "classic" problems in mathematics. This can be used to provide students with some of the history of mathematics and mathematicians.

Connecting Geometry Unit

Skills and Concepts	# of Days	Instructional Strategies	Assessment	Resources Needed	NCTM: Geometry
Experimental and Theorhetical Probability	1-2	Dizzy Darts Activity	Class Discussion	Tangrams Color Pencils Paper	Visualization Spacial Reasoning Geometric Modeli
Nth Term Problems Polygons Number Sense	1-2	Handshake Problem Triangular Numbers Handout	Open Response	Hands Dry Erase Board	Developing Math Arguments about Geo. Relationship
Pythagorean Thm Pythagorean Triples	1	The Egyptian Farmer Activity	Class Discussion	Rope 200 ft.	Pythagorean Relationships
Mobius Strips Scale	1	Atomic Origami Activity	Class Discussion	Paper Pencils Tape	Inductive Argume Describing size us scale
Introduction to Discrete Math	1	Bridges of Konigsberg	Open Response	Paper Pencils	Using visual tools e.g. Networks

Dizzy Darts Activity

Topic/Question: In this activity, students explore both experimental and theoretical probability through area.

Core Content: Students will address the following NCTM standards:
1) use appropriate terminology to describe complementary and mutually exclusive events;
2) use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments,
3) compute probabilities for simple compound events

Objectives: When this experiment is coupled with class lecture to assist in the correct usage of probability concepts and terminology, students should be able to do all of the following:

1) recognize the use of experimental and theoretical probability
2) use area models as a way of understanding the probability of an event
3) talk about the differences between experimental and theoretical probability
4) use mathematical reasoning (not formulae) to find the theoretical probability of some events
5) use repeated trials to find the experimental probability of an event

Materials: Blank paper, Pencils, Tangrams (2 sets per pair), crayons (colored pencils, markers...), calculators (optional), and some "darts" (beans, pennies...), anything that will fall and stay on the dartboard.

Procedure/Time: The activity should only take one class period (about fifty minutes), and should require two hours of preparation. Some items are written in to the instructions which could easily expand the activity from one day to two.

Assessment: Following the activity the teacher should prompt the students with several questions, to gauge their understanding of probabilities involved in the experiment.
Some examples:
Does moving the tangram shapes around change the theoretical probability? What do you expect to happen with increased trials in the experiment? If you know what the theoretical probability is of hitting one color, what is the probability of hitting any other color? If we get lucky and hit red three times in a row does this change our theoretical probability of hitting red a fourth time?...

Dizzy Darts Activity Instructions

<u>Class Discussion</u>

Have students answer these questions and write their responses on the board.

1. What are some “real life” events that involve probability?
2. What are some games that depend on Geometry?
3. Which items are sometimes used in the classroom to discuss probability?

Discuss at some length the relationships between specific answers to these questions. If an answer for #1 is Racing talk about distance. Question three helps bring up the subject of area, particularly answers like spinners or dice. A good question to ask might be, “why are dice shaped the way they are.” You can also talk about fairness this way by bringing up the subject of loaded dice, etc.

Have students describe the shape and structure of a dartboard and draw one on the board as you talk about how each aspect pertains to the subject of area in the context of probability. A dart thrown at random at the board has a small chance of hitting the bull’s-eye because the bull’s-eye is much smaller than the rest of the board…

It may help to have student experiment for some time with tangram shapes before they begin the activity but this will almost certainly make the activity two days.

<u>Begin Activity</u>

Have the students work in pairs. Each pair makes a dartboard by taking a blank sheet of paper and outlining tangram shapes one piece at a time until most of the center of the paper is filled. While one of the two is outlining the other can color the board in with colored pencils. Promote individuality while trying to keep to some of the following guidelines:

1. Stick to about 4 colors, while making sure adjacent shapes are different colors. Many students will figure out that it is possible to color the board with only two or three colors. Coloring the board with as few colors as possible relates to Discrete Mathematics.
2. Try to keep all the shapes close together; it makes results from playing the game easier to read.
3. Some students will work faster than others; stop them when about a third of the class has filled the entire page. This way the slower working student will only have about half the page filled. This will speed them up when it comes to playing the game and taking data.

Have a variety of materials ready for darts. These should be any small items that will fall and lay flat on the board. I found that pennies and coffee beans work well.

Experimental Probability

Students find this by repeated trials dropping the dart onto the board and recording the color it lands on. They should trade places regularly; one person being the dropper the other the recorder. Have them record data in their own way. Frequency charts work well. Begin with twenty drops and have them calculate the experimental probability as a fraction, percentage and decimal for **each color** on their board.

Experimental probability for **Blue** = The number of times the dart landed on **Blue** / The total number of drops

Have them do at least two trials with an increasing number of drops. I found it worked well to do around twenty for trial 1, then another twenty to make a total of forty drops for trial 2. You should talk with the students about how many drops will be sufficient and then agree upon some number for each trial.

It is essential that students label all results.

Theoretical Probability

The smallest triangular tan gram is used as a "unit." Have them find the theoretical probability of hitting each color.

Theoretical probability of hitting Blue = The number of unit triangles needed to cover Blue / The number of triangles needed to cover the board

End of Class Discussion

Use inquiry based teaching to discuss the relation ship between experimental and theoretical probability as it relates to area. Some good questions are:

- Does moving the tan gram shapes around change the experiment?
- What do you expect to happen with an increasing number of drops for trial 3, 4?
- Suppose you have two colors if you know what the theoretical probability of hitting one of them is, do you know the theoretical probability of hitting the other?
- If we hit the color red three times in a row does it change our chance of hitting red again?
- How can we make this game fair for two or three players?
- What is a sufficient number of drops for finding the experimental probability?

Handshake Problem Activity

Topic/Question: The main question is "if n people in a room must all shake hands, how many handshakes occur?" (which is answerable in geometric terms) however, the activity is geared towards learning problems solving. In particular how to solve nth term problems.

Core Content: Students will address the following NCTM standards for geometry 6-8:
1) create math arguments about geometric relationships;
2) develop inductive reasoning skills
3) use visual tools such as networks to represent and solve problems
4) apply geometric ideas in areas outside the math classroom

Objectives: Students should be able to do all of the following:
1) develop math arguments to solve nth term problems
2) recognize the use of networks in everyday life
3) further understand the use of counting arguments in mathematics

Materials: Blackboard or Dry Erase and students

Procedure/Time: The activity should only take one class period (about fifty minutes), and should require two hours of preparation. The activity can be expanded to two days, especially using discussion of the open response worksheet on triangular numbers (which are very closely related to the handshake problem).

Assessment: By class discussion following the activity exploring the concepts involved: counting, polygons, the variable n...

Handshake Problem Activity Instructions

Class Discussion

Have the students answer these questions and write their responses on the board.

1 How do we get from place to place?
2 How do we communicate with each other?
3 What are some ways we great one another?

Discuss at some length the mathematics involved with items listed in each answer. Show on the board how several items listed from these questions can be described as **Networks** (graphs). Talk about the use of this word and make connections with the students about what it means in this context. If Airplanes are listed for number one you can talk about why airplane routes utilize hub cities. If Computers or phones are listed for question two you can talk about why these systems use routers for communication instead of running lines from each persons home to everyone else's.

Show how these networks are simply n-gons with each of the diagonals drawn in.

Make a note of the differences between two-way communication/travel and one-way. Letters verses phone calls or Driving verses Train routes. (Graphs Vs. Directed Graphs)

Talk about Nth term problems bringing up any examples they may have looked at previously. Discuss, in particular, what mathematicians mean by the number n (natural numbers, variables…). This may take some time depending on pre-algebra skills.

Handshake Problem

First note that this activity is about learning **problem solving** not necessarily answering this particular question. It is more important that the students try several of their own methods rather than achieving the exact answer: n*(n-1)/2

Pose the question, "How many ways can n people shake hands?" any way you want, but make sure to phrase it carefully. You can also ask, "If n people are in a room and they all must shake hands with one another (exactly once) how many handshakes take place?"

The above in inquiry based mathematics terms is a "worthy mathematical task" and the students must find a solution.

Goals for solving this problem

- Have students make a table showing solutions for 1, 2, 3…(about) 12 people.
- Have them work in groups (actually shaking hands), and regroup as needed for larger solutions. Note that counting handshakes can be tricky so tell them if they are correct or not.
- Have them graph the problem.

If the students have trouble solving the problem this way (every one of my classes did), then move the problem to Japan, and discuss bowing.

Simplifying the problem

Look back at answers from question three for one that involves one way communication (email, letters, exchange of business cards, etc…), if bowing is not one that's been mentioned write it up there (or just prompt it and write it up there at the beginning of the class).

Discuss the differences between bowing to one another and shaking hands. Note that this simplifies the answer from n*(n-1)/2 to just n*(n-1).

Have students solve this problem in the same way as above. Note that if you haven't seen a classroom filled with 25-30 students all bowing to one another it's worth it just to see that.

Work as a class to find the solution n*(n-1), for the bowing problem using charts, tables, graphs, whatever it takes. Then note that [two bows = one handshake] and get to
n*(n-1)/2 as a solution for the handshake problem.

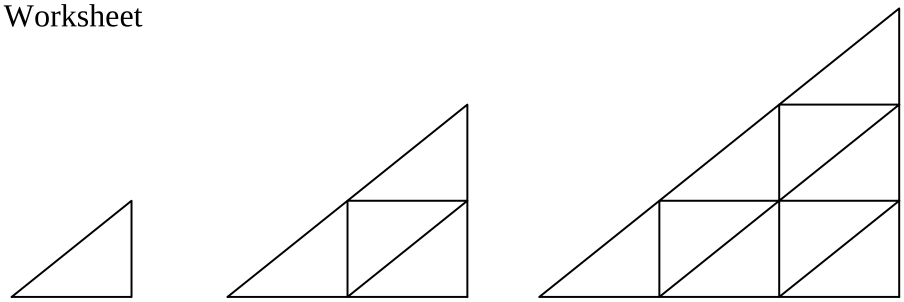
Connecting the Handshake Problem to Mathematics

- n*(n-1)/2 = the number of sides of an n-gon + the number of diagonals.
- n*(n-1)/2 = the combination of n things taken two at a time.
- n*(n-1)/2 is closely related to triangular numbers for which we achieve the expression (n+1)*n/2.

The open response on triangular numbers should be given as homework to be discussed the following day.

Feel free to give the following hint, shown on back of the worksheet.

Triangular Numbers
Open Response Worksheet



1 2 3 4

Examine figures 1, 2, 3, and 4 above.

Please describe any patterns you see in the Figures shown above.

How many dots will appear in Figure 5? Figure 6? Feel free to draw on the back of this sheet, or to use scrap paper.

Can you state, without drawing the picture, how many dots will appear in figure 10? If so, how many are there? Justify your response.

What Figure will have 78 dots?

Please provide an expression in terms of n, for the number of dots in Figure n.

The Egyptian Farming Activity

Topic/Question: How do we create a rectangular field with perfect right angles using only a length of rope?

Core Content: Students will address the following NCTM standards for geometry 6-8:
1) create math arguments concerning geometric relationships e.g. Pythagorean relationships
2) apply geometric ideas in areas outside the math classroom
3) apply transformations to analyze mathematical situations

Objectives: Students should be able to do all of the following:
1) understand and apply the Pythagorean theorem to areas of everyday life
2) understand and use Pythagorean triples

Materials: Blackboard or Dry Erase, pencil and paper, rope (several feet)

Procedure/Time: The activity should only take one class period (about fifty minutes), and should require two hours of preparation.

Assessment: By class discussion following the activity exploring the concepts involved.

Egyptian Farmer Activity Instructions

Review of the Pythagorean Theorem

Place seven or eight right triangles on the board. Label two sides of each triangle with integer lengths. Make sure that several give whole number solutions using the Pythagorean Theorem. These are Pythagorean triples. Try to use varying degrees of difficulty. To create triples select any two integers m and n then (m^2-n^2), $(2mn)$, and (m^2+n^2) are a triple. Have the students solve them and go over the problems as a group discussing the appearance of Pythagorean triples. Make sure to note how triangles with edges of lengths equal to any set of Pythagorean triples must itself be a right triangle. The lengths of the three sides uniquely determine a tringle.

The Activity

Find a large open area for the students to work e.g. a field. Put them in groups and present them with the task of staking out a large rectangular area with perfect right angles at the corners. Give each group one or two lengths of rope knotted at regular intervals with enough knots to create a right triangle using Pythagorean triples. That is, with twelve knots one may create a triangle with side lengths 3-4-5. This is then a right angle and can be used to help the students stake out a rectangular area. This was the method used by the ancient Egyptians for creating rectangular farms

The emphasis of this activity is problem solving and many of the student will come up with their own methods for stake out the required area.

Use one of the groups to demonstrate the above solution to the rest of the group and explain its historical significance. A field that has bleachers for seating the rest of the class can be useful.

Atomic Origami Activity

Topic/Question: Expanding the framework for students' concepts of the atom, the mathematics associated with its size, and a look Eastern thought relating to partical/wave duality

Core Content: Students will address the following NCTM standards for geometry 6-8:
1) use geometric models to represent and explain numerical and algebraic relationships
2) apply geometric ideas in areas outside the math classroom
3) describing size using scale
4) develop inductive arguments

Objectives: Students should be able to do all of the following:
1) explore the ideas concerning the atom through a new perspective
2) have an increased understanding of the use of numbers as a description of size

Materials: several sheets of blank paper, scissors and tape

Procedure/Time: The activity should only take one class period (about fifty minutes), and should require two hours of preparation.

Assessment: By class discussion following the activity exploring the concepts involved.

Atomic Origami

Class Discussion

Begin by asking this question and writing the students' descriptions on the board.

- What are atoms?

Use the students' interests and knowledge to promote discussion of the size of things and how this relates to science and mathematics. Make sure to note an appropriate definition to the class, "building blocks of matter" or however they are discussed in the science classes of the school.

Draw a line all the way across the chalkboard (dry erase, whatever) and place atoms on the far left as the extremely small. Ask the students for an idea for something big and place that on the right hand side (try to pick something between the size of a basketball and a classroom). Have the students name and place items on the line of intermediate sizes, i.e. frog, cells, bacteria, Kevin's right foot…

Ask if the placement is proportionally correct, and move things about until the class is mostly in agreement.

Switch gears and talk about the scientific mole. Give the definition 1 mole = 6.02*10^23 and write out all 21 zeroes. This is another way the student develop a context for the size of numbers. It is also a good way for going over scientific and standard notations.

Discuss how the term mole is similar to dozen. When shopping for eggs you buy them by the dozen. When a chemist shops for molecules he or she buys them by the mole. Why?

Begin the Scale Activity

Ask the student to imagine that a single letter size sheet of paper is actually a sheet consisting of 1 mole of carbon atoms. Fold, crease and tear the sheet in half, discarding one side and asking the students how many carbon atoms are left. Repeat a few times so the students see the pattern…half, fourth, eighth…

Have the students do this with their own sheet of paper, creating a discard pile consisting of iterative halves. It is quite possible to do this 13-15 times, although some students will claim 17-20, and you can't really see the paper well at that point. Ask the students how many times the paper must be folded in half until only one atom of carbon is left. Have the class come to some kind of agreement.

Take one of the discard piles and 13-14 volunteers from the classroom to line up in order showing the geometric progression of halves of paper. Discuss with the class that it would take 78 tears (in this context) before one is left with one atom of carbon. Talk about what this means in terms of the spacing of items lined up on the board.

Begin the Möbius Strip Activity

Have the students creates strips of paper around 2 inches wide and 11 or so long, and tape them together on the ends after a single twist in the paper. This is a möbius strip. Explore with the students the number of sides and edges (only one each). For many students this ends up being an irritating anomaly and should generate quite a bit of disagreement among classmates.

For higher level classes this activity can be used to prompt discussion of Eastern thought and can be related to the more advanced concepts of particle wave duality. The möbius strip is an object which seems to have both one and two surfaces simultaneously, while light exhibits characteristics of both waves and particles. Western thought on the other hand tends to think of things in terms of extrema: up or down, fine or coarse, hot or cold.

The Bridges of Konigsberg Activity

Topic/Question:

Core Content: Students will address the following NCTM standards for geometry 6-8:
1) use visual tools such as networks to represent and solve problems
2) apply geometric ideas in areas outside the math classroom

Objectives: Students should be able to do all of the following:
1) recognize and use graphs, networks to understand mathematical situations in the real world
2) develop mathematical arguments to solve problems

Materials: several sheets of blank paper, pencils

Procedure/Time: The activity should only take one class period (about fifty minutes), and should require two hours of preparation.

Assessment: By class discussion following the activity exploring the concepts involved.

The Bridges of Königsberg Activity Instructions

Begin Activity

Present the classic problem of the bridges of Königsberg to the students as it is shown on the attached worksheet. Feel free to add as much detail pertaining to history and geography as needed. It helps to do some research concerning the region as well as history of the problem and associated mathematics although these are not strictly necessary.

Have the students attempt to solve the problem with pencil and paper until most of the class is convinced that it cannot be done. Challenge the class to decide why the problem is unanswerable. The objective here is to get students to analyze a problem with a specific goal in mind, without providing algorithmic methods for deriving a solution.

It is essential that students document their observations and responses.

Looking at the problem from a new perspective

Within this unit students should already have been exposed to networks. Explain how the map provided is analogous a network. This can be tricky.

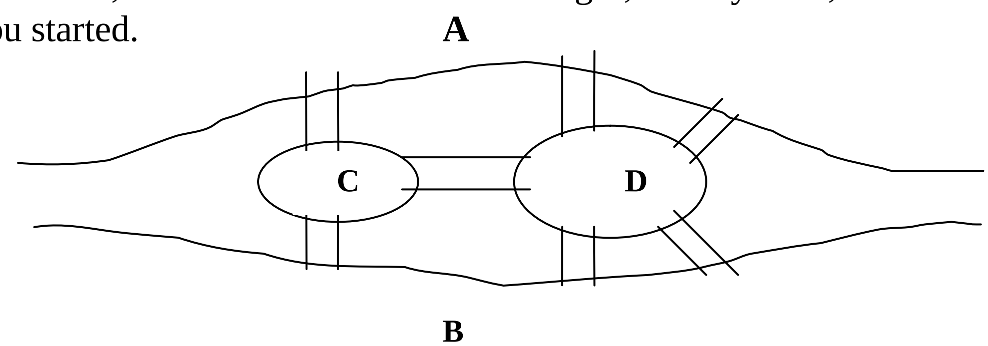
It helps to talk about how if a person is standing on either of the two islands they can walk wherever they want on those islands without crossing any bridges. Thus the two islands can be redrawn on our map shrinking them down to small dots a.k.a. vertices. Similarly the top and bottom land masses can be shrunken to dots resulting in network or graph where the bridges are the edges. The worksheet may be provided as a open response homework.

Provide additional graphs as needed to help students solve the problem. That is not every vertex has an even number of edges or simply, not every landmass has an even number of bridges coming out of it.

The Bridges of Königsberg

Part I

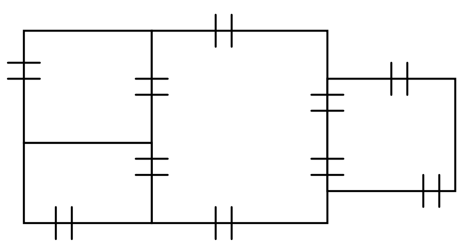
Look at the map below. Pick an area of land (A, B, C or D) and try to make a path that begins there, crossed each of the seven bridges, exactly once, and ends up back where you started.



You may try as many times as you like.
Decide if it is possible. State your reasons below.

Part II

Look at the layout of the house below. Make a path that begins in any of the rooms shown (or the outside) that goes through each door exactly once, and ends in the place you started.



Part III

What things are different about parts I and II?

What things are the same?